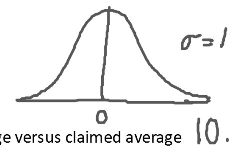
REVIEW:

Testing Proportions (percents): Testing sample percent versus claimed percent Ch.9

Test Statistic:

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

Distribution: Z - distrib (normal)

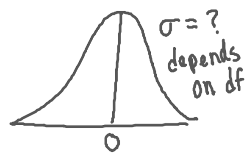


Testing Means (averages): Testing sample average versus claimed average 10.2

Test Statistic:

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Distribution: t distrib.



NEW: Testing full distributions:

Example: DICE

We roll a die 120 times and find the following outcomes: 18 ones, 19 twos, 17 threes, 22 fours, 20 fives, 24 sixes. Let's compare the observed to the expected:

outcomes

	1	2	3	4	5	6
sample Observed Values	18	19	17	22	20	24
Expected Value claims	20	20	20	20	20	20

EXAMPLE: M&M's

M&M's are claimed to have the following percentages: 24% blue, 14% brown, 16% green, 20% orange, 12% red, 14% yellow. We take a sample of 200 M&Ms and find the following: 45 blue, 33 brown, 30 green, 44 orange, 20 red, 28 yellow. Let's compare observed to expected:

	Blue	Brown	Green	Orange	Red	Yellow
Observed Values	45	33	30	44	20	28
Expected Values	48	28	32	40	24	28

Chi-Square Goodness of Fit Test

- Testing whether…

  An observed distribution (sample) fits an expected distribution (claim)

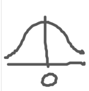
Hypothesis- different! WRITTEN OUT!

- Ho: the observed distribution of ~~dice~~ M&M's fits the expected distribution. (=)
- Ha: the observed distribution of ________ doesn't fit the expected distribution. (≠)

Test Statistic

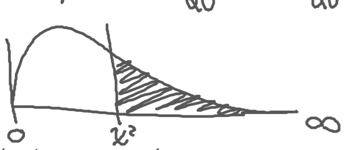
- Symbol: χ^2 Called: Chi- Square test
- Formula:

$$\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$



$$\chi^2 = \frac{(18-20)^2}{20} + \frac{(19-20)^2}{20} + \ldots$$

- Distribution: Chi-Square distribution



- df = # of categories/outcomes -- 1

P-value

- Work: $P(\chi^2 > \underline{\text{test statistic}}) = \text{p-value}$
- On calculator: $\chi^2\text{cdf}(LB, \overset{\infty}{UB}, df)$
- Always…. looking GREATER than the test statistic

Conclusion

- Still the same two sentences!
- We reject/fail to reject Ho b/c the p-value of _____ is </> alpha = _____.
- We have/do not have sufficient evidence that the ... (re-copy the Ha)

Conditions to check:

- SRS
- All ***EXPECTED*** numbers ≥ 5

Example #1

Portable personal computers, or "laptops," represent a fast-growing segment of the PC market. According to Market Intelligence Research company, the use of laptops can be classified in the following user segments ("Laptop's Three Musts," 1988): Business-professional (69%), Government (21%), Education (7%), and Home (3%). 150 laptop owners were surveyed this year, and the user segments were tabulated as follows: Business-professional (102), Government (32), Education (12), and Home (4). Do the data provide sufficient evidence to indicate that the figures given in 1988 are not accurate today?

	B/P	G	Ed	H
Obs.	102	32	12	4
Exp	103.5	31.5	10.5	4.5

Conditions:

1) SRS — 1) assume representative

2) all exp. ≥ 5 — 2) X Home < 5

Ho: the obs. distr. of laptop use fits the exp. distr.

Ha: the obs. distr. of laptop use does not fit the exp. distr.

$$\chi^2 = \frac{(102-103.5)^2}{103.5} + \frac{(32-31.5)^2}{31.5} + \frac{(12-10.5)^2}{10.5} + \frac{(4-4.5)^2}{4.5}$$

$$\chi^2 = 0.022 + 0.008 + 0.214 + 0.056$$

$$\chi^2 = 0.30$$

$$P(\chi^2 > 0.30) = 0.96$$

df = 3 $\qquad \alpha = 0.05$

- We do not reject the Ho b/c p-value of 0.96 > α = 0.05.
- We do not have evidence that the obs. distr. of laptop use does not fit the exp. distr.

Example #2:

A professor of education classes at Virginia Tech wants to look at what types of education the VT students are choosing. From previous studies, the types of education have been known to have the following distribution: 25% physical education, 15% math education, 15% science education, 5% art education, 20% special education, 10% history education, 5% foreign language education, and 5% other. He takes a random sample of 154 education majors and finds the following results: 40 phys ed, 20 math, 20 science, 10 art, 30 special ed, 15 history, 10 foreign language, and 9 other. Has the distribution of education majors changed? Run a full test of significance.

	PE	ME	SE	AE	SpE	H	FL	O
Obs	40	20	20	10	30	15	10	9
Exp	38.5	23.1	23.1	7.7	30.8	15.4	7.7	7.7

Conditions:

1) SRS — 1) stated random

2) all exp. ≥ 5 — 2) see table. all exp ≥ 5

CALCULATOR:

- Put observed into L1
- Put expected into L2
- L3 = (L1 -- L2)^2/L2
- 2ND --> LIST --> MATH --> sum(
- sum(L3) = χ^2

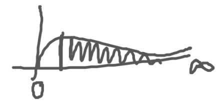
Ho: the obs. distr. of ed. majors fits the exp. distr.

Ha: the obs. distr. of ed. majors does not fit the exp.

df=7

$$\chi^2 = \frac{(40-38.5)^2}{38.5} + \frac{(20-23.1)^2}{23.1} + \ldots = 2.515$$

$P(\chi^2 > 2.515) = 0.926$



- fail to reject Ho....
- Do not have evid.... (Ha)

Worksheet:

1. A grocery store manager wishes to determine whether a certain product will sell equally well in any of five locations in the store. Five displays are set up, and the resulting numbers of the product sold are 43, 29, 52, 34, and 48. Is there enough evidence that the location makes a difference? Test at the 5% significance level.

Location	#1	#2	#3	#4	#5
Observed	43	29	52	34	48
Expected	41.2	41.2	41.2	41.2	41.2

Ho: The observed distribution of locations fits the expected distribution (it is uniform)

Ha: The observed distribution of locations doesn't fit the expected distribution (it is not uniform)

STATE	CHECK
1- SRS	1- assumed representative
2- all expected #'s $\geq$ 5	2- see chart above

$$\chi^2 = \Sigma \frac{(obs -- exp)^2}{exp} = \frac{(43 -- 41.2)^2}{41.2} + \frac{(29 -- 41.2)^2}{41.2} + \ldots$$

$\chi^2 = 8.903$

$P(\chi^2 > 8.903) = 0.0636$ df = 4

We fail to reject Ho b/c p-value of 0.0636 > α = 0.05.
We do not have sufficient evidence that the observed distribution of locations is not uniform.

2. An SRS of 581 coffee drinkers asks them where they typically drink their first cup of coffee each day. The results were as follows: 389 at home, 110 at work, 55 while commuting, and 27 at a restaurant/coffee bar/other. From previous surveys, it has been claimed that the following percents were true: 70% at home, 17% at work, 8% while commuting, and 5% at a restaurant/coffee bar/other. Is there sufficient evidence that there has been a change in coffee drinking?

2. An SRS of 581 coffee drinkers asks them where they typically drink their first cup of coffee each day. The results were as follows: 389 at home, 110 at work, 55 while commuting, and 27 at a restaurant/coffee bar/other. From previous surveys, it has been claimed that the following percents were true: 70% at home, 17% at work, 8% while commuting, and 5% at a restaurant/coffee bar/other. Is there sufficient evidence that there has been a change in coffee drinking?

	H	W	C	R/B/C
Observed	389	110	55	27
Expected	406.7	98.77	46.48	29.05

Ho: The observed distribution of first cups of coffee fits the expected distribution

Ha: The observed distribution of first cups of coffee doesn't fit the expected distribution

STATE
1- SRS
2- all expected #'s ≥ 5

CHECK
1- stated
2- see chart above, all exp > 5

$\chi^2 = \Sigma \frac{(obs - exp)^2}{exp} = \frac{(389 - 406.7)^2}{406.7} + \frac{(110 - 98.77)^2}{98.77} + \ldots$

$\chi^2 = 3.7536$

$\chi^2 cdf(3.7536, E99, 3)$

$P(\chi^2 > 3.7536) = 0.2893$ df = 3

We fail to reject Ho b/c p-value of $0.2893 > \alpha = 0.05$.
We do not have sufficient evidence that the observed distribution of first cups of coffee does not fit the expected distribution.

3. A program for generating random numbers on a computer is to be tested. The program is instructed to generate 100 single-digit integers between 0 and 9. The frequencies observed are 11, 8, 7, 7, 10, 10, 8, 11, 14, and 14. Is there sufficient reason to believe that the integers are not being generated **uniformly**?

3. A program for generating random numbers on a computer is to be tested. The program is instructed to generate 100 single-digit integers between 0 and 9. The frequencies observed are 11, 8, 7, 7, 10, 10, 8, 11, 14, and 14. Is there sufficient reason to believe that the integers are not being generated **uniformly**?

	0	1	2	3	4	5	6	7	8	9
Obs	11	8	7	7	10	10	8	11	14	14
Exp	10	10	10	10	10	10	10	10	10	10

Ho: The observed distribution of generated numbers is uniform

Ha: The observed distribution of generated numbers is not uniform

STATE
1- SRS
2- all expected #'s ≥ 5

CHECK
1- stated
2- see chart above

$\chi^2 = \Sigma \frac{(obs - exp)^2}{exp} = \frac{(11 - 10)^2}{10} + \frac{(8 - 10)^2}{10} + \ldots$

$\chi^2 = 6$

$P(\chi^2 > 6) = 0.7399$ df = 9

We fail to reject Ho b/c p-value of $0.7399 > \alpha = 0.05$.
We do not have sufficient evidence that the observed distribution of generated numbers is not uniform.

4. The following chart shows the distribution of the time of day of roadside crash deaths from the previous year:

time	6 am - noon	noon - 6pm	6 pm - midnight	midnight - 6 am
deaths	15%	22%	29%	34%

An SRS of 627 roadside crash deaths showed the following results:

time	6 am - noon	noon - 6pm	6 pm - midnight	midnight - 6 am
deaths	128	115	160	224

At α=0.01, test to see if the survey shows that the distribution has changed.

4. The following chart shows the distribution of the time of day of roadside crash deaths from the previous year:

time	6 am - noon	noon - 6pm	6 pm - midnight	midnight - 6 am
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An SRS of 627 roadside crash deaths showed the following results:

time	6 am - noon	noon - 6pm	6 pm - midnight	midnight - 6 am
deaths	128	115	160	224

At α=0.01, test to see if the survey shows that the distribution has changed.

OBS	128	115	160	224
EXP	94.05	137.94	181.83	213.18

Ho: the obs. distr. of times of day of crashes fits the exp. distr.
Ha: the obs. distr. of times of day of crashes doesn't fit the exp. distr.

STATE
1- SRS
2- all expected #'s $\geq$ 5

CHECK
1- stated
2- see chart above

$$\chi^2 = \Sigma \frac{(obs - exp)^2}{exp} = \frac{(128 - 94.05)^2}{94.05} + \frac{(115 - 137.94)^2}{137.94} + \ldots$$

$\chi^2 = 19.24$

$P(\chi^2 > 19.24) = 2.4384 \times 10^{-4}$ df = 3

We reject Ho b/c p-value of $2.4384 \times 10^{-4} < \alpha = 0.05$.
We have sufficient evidence that the observed distribution of times of day of crashes does not fit the expected distribution.